CORRECTION TO: ON THE CHARACTERIZATION OF NONRADIATING SOURCES FOR THE ELASTIC WAVES IN ANISOTROPIC INHOMOGENEOUS MEDIA

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The purpose of this note is to address typographical errors in [KW21, Lemma 4.2 and Appendix B].

In the proof of [KW21, Lemma 4.2], the function $\mathbf{w}$ should be taken as incoming, which is equivalent to $\overline{\mathbf{w}}$ being outgoing. More precisely, the expression

"$\mathbf{w}$ satisfies the Kupradze radiation condition at $|\mathbf{x}| \to \infty$"

should be replaced with

"$\overline{\mathbf{w}}$ satisfies the Kupradze radiation condition at $|\mathbf{x}| \to \infty$".

We do not repeat the proof here, but recall that it relies on the following fact:

$$\lim_{R\to\infty} \iint_{\partial B_R} \left(\mathcal{B}_{\hat{\mathbf{x}}}\mathbf{v} \cdot \overline{\mathbf{w}} - \mathbf{v} \cdot \overline{\mathcal{B}_{\hat{\mathbf{x}}}\mathbf{w}} \right) \mathrm{d}s(\mathbf{x}) = 0, \tag{1}$$

where the traction on ∂B_R is given by

$$\begin{aligned} \mathcal{B}_{\hat{\mathbf{x}}}\mathbf{v} &= (\mathbb{C} : \nabla \mathbf{v})\nu = (\mathbb{C} : \nabla \mathbf{v})\hat{\mathbf{x}} \\ &= 2\mu\hat{\mathbf{x}} \cdot \nabla \mathbf{v} + \lambda\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v} + \mu\hat{\mathbf{x}} \times \mathrm{curl}\,\mathbf{v} \\ &= 2\mu\partial_{|\mathbf{x}|}\mathbf{v} + \lambda\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v} + \mu\hat{\mathbf{x}} \times \mathrm{curl}\,\mathbf{v}. \end{aligned} \tag{2}$$

We also remark that the Green's formula for radiating elastic wave, given in [CKA+07, equation (11)], involves the same limit as in (1), with $\overline{\mathbf{w}(\mathbf{x})} = \Gamma(\mathbf{x} - \mathbf{y})$. This limit can be justified using the Kupradze radiation condition, see Appendix A for details.

Additionally, the function $\mathbf{w}$ in [KW21, Appendix B] should be compactly supported. This correction does not affect the subsequent discussions. We remind readers that the compact support of $\mathbf{w}$ is necessary to ensure

$$\lim_{R\to\infty} \iint_{\partial B_R} \mathcal{B}_{\hat{\mathbf{x}}}\mathbf{u} \cdot \overline{\mathbf{w}}\, \mathrm{d}s(\mathbf{x}) = 0$$

in the proof of [KW21, Lemma B.1]. In the first equation of [KW21, (B.1)], $\Omega \setminus \Sigma$ should be replaced by $\mathbf{R}^3 \setminus \Sigma$.

Appendix A. Traction operator on sphere and radiation condition

Let $\mathbf{v}$ satisfies $(\mathcal{L}^{\lambda,\mu} + \omega^2)\mathbf{v} = 0$ in $\mathbf{R}^3 \setminus \overline{\Omega}$, and write $\mathbf{v} = \mathbf{v}^{(p)} + \mathbf{v}^{(s)}$ as in [KW21, Lemma 2.1]. Assume that such $\mathbf{v}$ satisfies the Kupradze radiation condition as defined in [KW21, Definition 2.2]. By applying the cross product rule, we obtain

$$\hat{\mathbf{x}} \times \mathrm{curl}\,\mathbf{u} = \hat{\mathbf{x}}\mathrm{div}\,\mathbf{u} - \hat{\mathbf{x}} \cdot \nabla \mathbf{u} \quad \left(\equiv \hat{\mathbf{x}}\mathrm{div}\,\mathbf{u} - \partial_{|\mathbf{x}|}\mathbf{u} \right)$$

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for all smooth $\mathbf{u}$. By choosing $\mathbf{u} = \mathbf{v}^{(p)}, \mathbf{v}^{(s)}$, we know that

$$\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v}^{(p)} = \partial_{|\mathbf{x}|}\mathbf{v}^{(p)} = o(1), \quad \hat{\mathbf{x}} \times \mathrm{curl}\,\mathbf{v}^{(s)} = -\partial_{|\mathbf{x}|}\mathbf{v}^{(s)} = o(1). \tag{3}$$

Thus, the traction $\mathcal{B}_{\hat{\mathbf{x}}}$ on ∂B_R, as given in (2), is written as

$$\begin{aligned} \mathcal{B}_{\hat{\mathbf{x}}}\mathbf{v} &= 2\mu\partial_{|\mathbf{x}|}\mathbf{v} + \lambda\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v} + \mu\hat{\mathbf{x}} \times \mathrm{curl}\,\mathbf{v} \\ &= 2\mu\partial_{|\mathbf{x}|}\mathbf{v}^{(p)} + 2\mu\partial_{|\mathbf{x}|}\mathbf{v}^{(s)} + \lambda\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v}^{(p)} + \mu\hat{\mathbf{x}} \times \mathrm{curl}\,\mathbf{v}^{(s)} \\ &= (\lambda + 2\mu)\partial_{|\mathbf{x}|}\mathbf{v}^{(p)} + \mu\partial_{|\mathbf{x}|}\mathbf{v}^{(s)} \quad (\text{using } (3)) \\ &\equiv (\lambda + 2\mu)\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v}^{(p)} + \mu\mathrm{curl}\,\mathbf{v}^{(s)} \times \hat{\mathbf{x}}. \end{aligned}$$

We now assume that $\overline{\mathbf{w}} = \overline{\mathbf{w}}^{(p)} + \overline{\mathbf{w}}^{(s)}$ is the function in (1). By combining the Kupradze radiation condition with (3), we obtain

$$\begin{aligned} \overline{\mathbf{w}}^{(p)} &= \frac{1}{ik_p}\hat{\mathbf{x}}\mathrm{div}\,\overline{\mathbf{w}}^{(p)} + o(|\mathbf{x}|^{-1}), \\ \overline{\mathbf{w}}^{(s)} &= \frac{1}{ik_s}\mathrm{curl}\,\mathbf{v}^{(s)} \times \hat{\mathbf{x}} + o(|\mathbf{x}|^{-1}). \end{aligned}$$

Therefore, on ∂B_R, we know that

$$\begin{aligned} &\mathcal{B}_{\hat{\mathbf{x}}}\mathbf{v} \cdot \overline{\mathbf{w}} \\ &= \left((\lambda + 2\mu)\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v}^{(p)} + \mu\mathrm{curl}\,\mathbf{v}^{(s)} \times \hat{\mathbf{x}}\right) \cdot \left(\frac{1}{ik_p}\hat{\mathbf{x}}\mathrm{div}\,\overline{\mathbf{w}}^{(p)} + \frac{1}{ik_s}\mathrm{curl}\,\mathbf{v}^{(s)} \times \hat{\mathbf{x}}\right) + o(R^{-1}) \\ &= (\lambda + 2\mu)\hat{\mathbf{x}}\mathrm{div}\,\mathbf{v}^{(p)} \cdot \frac{1}{ik_p}\hat{\mathbf{x}}\mathrm{div}\,\overline{\mathbf{w}}^{(p)} + \mu\mathrm{curl}\,\mathbf{v}^{(s)} \times \hat{\mathbf{x}} \cdot \frac{1}{ik_s}\mathrm{curl}\,\mathbf{v}^{(s)} \times \hat{\mathbf{x}} + o(R^{-1}) \\ &= (\lambda + 2\mu)\partial_{|\mathbf{x}|}\mathbf{v}^{(p)} \cdot \frac{1}{ik_p}\partial_{|\mathbf{x}|}\overline{\mathbf{w}}^{(p)} + \mu\partial_{|\mathbf{x}|}\mathbf{v}^{(s)} \cdot \frac{1}{ik_s}\partial_{|\mathbf{x}|}\overline{\mathbf{w}}^{(s)} + o(R^{-1}) \quad (\text{using } (3)) \\ &= (\lambda + 2\mu)ik_p\mathbf{v}^{(p)} \cdot \overline{\mathbf{w}}^{(p)} + \mu ik_s\mathbf{v}^{(s)} \cdot \overline{\mathbf{w}}^{(s)} + o(R^{-2}) \quad (\text{Kupradze radiation condtion}). \end{aligned}$$

Performing similar computations to those for $\mathbf{v} \cdot \mathcal{B}_{\hat{\mathbf{x}}}\overline{\mathbf{w}}$, we obtain

$$\mathcal{B}_{\hat{\mathbf{x}}}\mathbf{v} \cdot \overline{\mathbf{w}} - \mathbf{v} \cdot \overline{\mathcal{B}_{\hat{\mathbf{x}}}\mathbf{w}} = o(R^{-2}) \quad \text{on } \partial B_R,$$

which results in (1).

References

[CKA+07] A. Charalambopoulos, A. Kirsch, K. A. Anagnostopoulos, D. Gintides, and K. Kiriaki. The factorization method in inverse elastic scattering from penetrable bodies. *Inverse Problems*, 23(1):27–51, 2007. MR2302961, Zbl:1111.74024, doi:10.1088/0266-5611/23/1/002.

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