COMPLEX ANALYSIS (701026001, 112-1) - HOMEWORK 2

Return to TA by: October 3, 2023 (Tuesday) 16:00

Total marks: 50

Exercise 1. (10 points) Show that for any integers a, b, c, d, we can find integers u, v such that

$$(a^2 + b^2)(c^2 + d^2) = u^2 + v^2.$$

Exercise 2. (5+5 points)

(a) Prove $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$ for all $z_1, z_2 \in \mathbb{C}$.

(b) Let P be a nonconstant polynomial in z. Show that $|P(z)| \to \infty$ as $|z| \to \infty$.

Exercise 3. (10 points) Show that the function $f(z) = z\overline{z}$ is differentiable at $z = 0$, but not analytic near $z = 0$.

Exercise 4. (10 points) Prove Lemma 2.1.5 and verify Example 2.1.6 in the lecture note.

Exercise 5. (5+5 points)

(a) Suppose $f(z)$ is real-valued and differentiable for all real z. Show that $f'(z)$ is also real-valued for real z.

(b) Suppose $f(z)$ is real-valued and differentiable for all imaginary points z. Show that $f'(z)$ is imaginary at all imaginary points z.