# COMPLEX ANALYSIS (701026001, 112-1) - HOMEWORK 3

Return to TA by: October 12, 2023 (Thursday) 16:00

Total marks: 50

**Exercise 1.** (10 points) Show that the radius of convergence of $\sum_{n=1}^{\infty} nz^n$ is $R = 1$, and the series also diverges for $|z| = 1$.

**Exercise 2.** (10 points) Show that the radius of convergence of $\sum_{n=1}^{\infty} n^{-2} z^n$ is $R = 1$, and the series also converges for $|z| = 1$.

**Exercise 3.** (10 points) Show that the radius of convergence of $\sum_{n=1}^{\infty} n^{-1} z^n$ is $R = 1$. In addition, show that the series converges for all $z \in \partial B_1 \setminus \{1\}$ but diverges at $z = 1$.

**Exercise 4.** (10 points) Show that $\sum_{n=0}^{\infty} (z^n/n!)$ converges for all $z \in \mathbb{C}$.

**Exercise 5.** (10 points) Show that if $f(z) = \sum_{n=0}^{\infty} C_n z^n$ has a nonzero radius of convergence, then

$$C_n = \frac{f^{(n)}(0)}{n!} \quad \text{for all } n = 0, 1, 2, \cdots$$

where $f^{(n)}$ is the $n^{\text{th}}$ complex derivative of $f$. Show that for each $n = 0, 1, 2, \cdots$ that

$$f^{(n)}(z) = n! C_n + (n+1)! C_{n+1} z + \frac{(n+2)!}{2!} C_{n+2} z^2 + \cdots$$

for all $z$ in the domain of convergence.

**Announcement.** I have updated the lecture note to fix the mistakes in the statement and the proof of Theorem 2.2.11 (uniqueness of power series).