COMPLEX ANALYSIS (701026001, 112-1) - HOMEWORK 7

Return to TA by: December 5, 2023 (Tuesday) 16:00

Total marks: 50

Exercise 1 (10 points)**.** Find the power series expansion of $f(z) = z^2$ around $z = 2$.

Exercise 2 (10 points)**.** Find the power series expansion for e^z around any point $a \in \mathbb{C}$.

Exercise 3 (10 points)**.** Show that $\cos z$ is unbounded on $\mathbb{C}$, and solve the equation $\cos z = 2$.

Exercise 4 (5+5 points)**.** Suppose that P is an analytic polynomial of *odd* order with real coefficients, that is, $P(z) = \sum_{k=0}^{n} c_k z^k$ for some $c_k \in \mathbb{R}$ and an *odd* positive integer n.

(a) Show that there exists $z_0 \in \mathbb{R}$ such that $P(z_0) = 0$. (Hint: consider the conjugate of zeros of P)

(b) Show that P is equal to product of polynomials of the form either $a_1 z + a_0$ (for some $a_1, a_2 \in \mathbb{R}$) or $b_2 z^2 + b_1 z + b_0$ (for some $b_1, b_2, b_3 \in \mathbb{R}$).

Exercise 5 (10 points)**.** Let P be an analytic polynomial. Show that α is a zero of multiplicity $k \in \mathbb{N}$ if and only if

$$P(\alpha) = P'(\alpha) = \cdots = P^{(k-1)}(\alpha) = 0 \text{ and } P^{(k)}(\alpha) \neq 0.$$