COMPLEX ANALYSIS (701026001, 112-1) - HOMEWORK 8

Return to TA by: December 12, 2023 (Tuesday) 16:00

Total marks: 50 (10 bonus)

Exercise 1 (10 points)**.** Suppose that f has an isolated singularity at z_0. If $\lim_{z\to z_0}|f(z)| = +\infty$, show that f has a pole at z_0.

Exercise 2 (10 points)**.** Suppose f is analytic in $A_{0,\infty} \equiv \mathbb{C} \setminus \{0\}$ and satisfies $|f(z)| \leq \sqrt{|z|} + 1/\sqrt{|z|}$ for all $z \neq 0$. Show that f is a constant.

Exercise 3 (10 points)**.** Suppose f and g are entire functions with $|f(z)| \leq |g(z)|$ for all $z \in \mathbb{C}$. Prove that there exists a constant $c \in \mathbb{C}$ such that $f(z) = cg(z)$ for all $z \in \mathbb{C}$. [Hint: Think about why the zeros of entire functions are all isolated]

Exercise 4 (10+10+10 points)**.** Find the Laurent expansion for

(1) $\frac{1}{z^4+z^2}$ about $z = 0$.
(2) $\frac{\exp(1/z^2)}{z-1}$ about $z = 0$.
(3) $\frac{1}{z^2-4}$ about $z = 2$.