COMPLEX ANALYSIS (701026001, 113-1) - HOMEWORK 3

Return to TA by: October 18, 2024 (Friday) 12:00

Total marks: 50

Exercise 1 (10 points)**.** Show that the function $f(z) = z\overline{z}$ is differentiable *at* $z = 0$, but not analytic *near* $z = 0$. (**Note.** I already clarify the difference of “at” and “near” in the class.)

Exercise 2 (10 points)**.** Let $P(z) = \alpha_0 + \alpha_1 z + \cdots + \alpha_N z^N$ for all $z \in \mathbb{C}$. Show that P is entire with

$$P'(z) = \alpha_1 + 2\alpha_2 z + \cdots + N\alpha_N z^{N-1} \quad \text{for all } z \in \mathbb{C}.$$

(**Hint.** One way to prove this is using the binomial theorem)

Exercise 3 (10 points)**.** Suppose that g is the inverse of f near z_0 and that g is continuous there. If f is (complex) differentiable at $g(z_0)$ and if $f'(g(z_0)) \neq 0$, then g is differentiable at z_0 and

$$g'(z_0) = \frac{1}{f'(g(z_0))}.$$

Exercise 4 (10 points)**.** Find all entire function $f = u + \mathbf{i}v$ with $u(x, y) = x^2 - y^2$ for all $x, y \in \mathbb{R}$. (**Note.** It is not difficult to find an example of f. The most important thing is that you have to show that there is no any other candidate.)

Exercise 5 (10 points)**.** Show that there are no entire function $f = u + \mathbf{i}v$ with $u(x, y) = x^2 + y^2$ for all $x, y \in \mathbb{R}$.