COMPLEX ANALYSIS (701026001, 113-1) - HOMEWORK 5

Return to TA by: November 1, 2024 (Friday) 12:00

Total marks: 50

Exercise 1 (5+5+5 points)**.**

(a) Prove that $e^{z_1+z_2} = e^{z_1}e^{z_2}$ for all $z_1, z_2 \in \mathbb{C}$.
(b) For each $n \in \mathbb{N}$, show that $(\cos\theta + \mathbf{i}\sin\theta)^n = \cos(n\theta) + \mathbf{i}\sin(n\theta)$ for all $\theta \in \mathbb{R}$.
(c) Compute $\cos(\theta_1 + \theta_2 + \theta_3)$ and $\sin(\theta_1 + \theta_2 + \theta_3)$ for all $\theta_1, \theta_2, \theta_3 \in \mathbb{R}$ in terms of $\cos\theta_1, \cos\theta_2, \cos\theta_3, \sin\theta_1, \sin\theta_2$ and $\sin\theta_3$.

Exercise 2 (15 points)**.** Find ***all*** solutions for the equation $\sin z = 2$. (**Note.** There are infinitely many solutions for this equation.)

Exercise 3 (5 points)**.** Evaluate $\int_{\mathcal{C}} f$ where $f(z) = x^2 + \mathbf{i}y^2$ and

$$\mathcal{C} = \left[\ z(t) = t^2 + \mathbf{i}t^2 \;\middle|\; 0 \le t \le 1\ \right].$$

Here we denote $z = x + \mathbf{i}y$.

Exercise 4 (5 points)**.** Evaluate $\int_{\mathcal{C}} f$ where $f(z) = 1/z$ and

$$\mathcal{C} = \left[\ z(t) = \cos t + \mathbf{i}\sin t \;\middle|\; 0 \le t \le 2\pi\ \right].$$

(**Hint.** Which assumption in Cauchy closed curve theorem does not satisfied by this example?)

Exercise 5 (5+5 points)**.** Evaluate $\int_0^{\mathbf{i}} e^z \,\mathrm{d}z$ and $\int_{\pi/2}^{\pi/2+\mathbf{i}} \cos 2z \,\mathrm{d}z$. Here the integrals are defined in the proof of Theorem 3.2.9 in the lecture note.