COMPLEX ANALYSIS (701026001, 113-1) - HOMEWORK 7

Return to TA by: December 13, 2024 (Friday) 12:00

Total marks: 50

Exercise 1 (10 points)**.** Prove Theorem 5.4.1 (First binomial coefficient integral) by using Residue theorem (Theorem 5.3.6) and evaluation the residues via complex differentiation (Proposition 5.3.2).

Exercise 2 (10 points)**.** Prove Theorem 5.4.7 (Exponential integral) by using the Residue theorem (Theorem 5.3.6).

Exercise 3 (10 points)**.** Suppose that f has an isolated singularity at z_0. If $\lim_{z\to z_0} |f(z)| = +\infty$, show that f has a pole at z_0.

Exercise 4 (10 points)**.** Suppose f is analytic in $A_{0,\infty} \equiv \mathbb{C} \setminus \{0\}$ and satisfies $|f(z)| \leq \sqrt{|z|} + 1/\sqrt{|z|}$ for all $z \neq 0$. Show that f is a constant.

Exercise 5 (10 points)**.** Suppose f and g are entire functions with $|f(z)| \leq |g(z)|$ for all $z \in \mathbb{C}$. Prove that there exists a constant $c \in \mathbb{C}$ such that $f(z) = cg(z)$ for all $z \in \mathbb{C}$. [Hint: Think about why the zeros of entire functions are all isolated]