FOURIER ANALYSIS (MATS315) - EXERCISE 3

Return by: September 30, 2022 (Friday)

Exercise 1 (Exercise 2.3.1, 10 pts)**.** Let $\phi_n(x) = e^{-\frac{1}{2}|x|^2}$, which is in $\mathscr{S}(\mathbb{R}^n)$ by Exercise 2.2.3 (suppose to be solved in Exercise 2). Prove that $\hat{\phi}_n = (2\pi)^{\frac{n}{2}}\phi_n$ and $\phi_n(0) = (2\pi)^{-n}\int_{\mathbb{R}^n}\hat{\phi}_n(x)\,dx$.

Exercise 2 (Exercise 2.3.5, 10 pts)**.** Prove Proposition 2.3.3.

Exercise 3 (Exercise 2.3.9, 10 pts)**.** Prove Proposition 2.3.8.

Exercise 4 (Exercise 2.4.4, 10 pts)**.** Prove that for each $a \in (-n, \infty)$ the function $|x|^a$ is a tempered distribution in $\mathbb{R}^n$. Therefore, we know that $\mathscr{O}_{\mathrm{M}}(\mathbb{R}^n) \subsetneq \mathscr{S}'(\mathbb{R}^n)$.

Exercise 5 (Exercise 2.4.9, 10 pts)**.** Prove Lemma 2.4.8.