

FOURIER ANALYSIS (MATS315) - EXERCISE 5

Return by: October 14, 2022 (Friday)

Exercise 1 (Exercise 2.14.3, 10 pts). Prove (2.14.2) in the proof of Theorem 2.14.1 (Structure of $\mathcal{D}'(\mathbb{R}^n)$ restricted on a bounded open set).

Exercise 2 (Exercise 2.16.2, 10 pts). Show that $\text{pv } \frac{1}{x} \in \mathcal{S}'(\mathbb{R})$.

Exercise 3 (Exercise 2.16.3, 10 pts). Prove that $(\ln|x|)' = \text{pv } \frac{1}{x}$ in $\mathcal{S}'(\mathbb{R})$.

Exercise 4 (Exercise 2.16.5, 10 pts). Prove that if $f \in C^0(\mathbb{R}^n \setminus \{0\})$ is positive homogeneous of degree $k \in \mathbb{R}$, then

$$|f(x)| \leq \|f\|_{L^\infty(\mathcal{S}^{n-1})} |x|^k \quad \text{for all } x \in \mathbb{R}^n \setminus \{0\}.$$

Exercise 5 (Exercise 2.16.12, 10 pts). Show that $T_{11} = f_1 * f_2$ where $f_j = (1 - \psi)\Theta_j$ for $j = 1, 2$, are functions belonging in $L^2(\mathbb{R}^n)$. Here Θ_1, Θ_2 are functions satisfy (2.16.3).

FINAL REPORT AND ORAL PRESENTATION

There is no final exam for this course. However it is necessarily to write a

write a final report [return by October 7, 2022 (Friday)]

and

giving an oral presentation

to obtain the credit. The final report no more than 10 pages in L^AT_EX (it is OK that if you still prefer to complete it by hand writing). Please return the final report before oral presentation. You only need to prepare about 10–15 minutes presentation. **Please let me know if you wish to give a longer presentation.**