GEOMETRY (701939001, 751764001, 113-2) - HOMEWORK 1

Return to TA by: March 4, 2025 (Tuesday) 16:00

Total marks: 50

Exercise 1 (10 points)**.** Sketch the functions $\sin^{-1} \circ \sin : \mathbb{R} \to \mathbb{R}$ and $\cos^{-1} \circ \cos : \mathbb{R} \to \mathbb{R}$.

Exercise 2 (20 points)**.** Prove the product-to-sum formulas (there are 4 in totals) using the methods in Remark 1.2.5, which involving the imaginary number.

Exercise 3 (10 points)**.** Compute $\frac{\mathrm{d}}{\mathrm{d}y}(\sin^{-1}(y))$ and $\frac{\mathrm{d}}{\mathrm{d}y}(\cos^{-1}(y))$ for all $y \in (-1, 1)$.

Exercise 4 (10 points)**.** For each $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$, we define the *juxtaposition* $\boldsymbol{u} \otimes \boldsymbol{v} := \boldsymbol{u}\boldsymbol{v}^\intercal \in \mathbb{R}^{n \times n}$. We consider the matrix $A := \mathrm{Id} + \boldsymbol{u} \otimes \boldsymbol{v}$, which is called the *rank-one perturbation of identity*. Determine the relation between $\boldsymbol{u}$ and $\boldsymbol{v}$ to guarantee A^{-1} exists, and compute A^{-1}.