GEOMETRY (701939001, 751764001, 113-2) - HOMEWORK 2

Return to TA **at March 18, 2025 (Tuesday) (13:00–13:10 or 15:00–15:10, TA's office)**

Total marks: 50

Exercise 1 (10+10 points)**.** Let $-\infty < a_k < b_k < +\infty$ for $k = 1, \cdots, n$.

(a) Show that the open interval $\prod_{k=1}^{n}(a_k, b_k)$ is an open set, but not a topological closed set, in $\mathbb{R}^n$.

(b) Show that the closed interval $\prod_{k=1}^{n}[a_k, b_k]$ is a topological closed set, but not an open set, in $\mathbb{R}^n$.

Exercise 2 (10+10+10 points)**.** We define the set $S := \{(x, f(x)) : x > 0\}$, called the *graph* of f on $(0, \infty)$. Determine the set of limit points S' (also known as *derived set*) when

$$\text{(a) } f(x) = x \sin \frac{1}{x}, \quad \text{(b) } f(x) = \sin \frac{1}{x}, \quad \text{(c) } \frac{1}{x} \sin \frac{1}{x}.$$

You will not get any credit if you just write down S' without any explanations.