GEOMETRY (701939001, 751764001, 113-2) - HOMEWORK 4

Return to TA by: April 1, 2025 (Tuesday) 16:00

Total marks: 50

Exercise 1 (10 points)**.** Let $S = S_1 \cup S_2$ where

$$S_1 = \left\{(x, y) \in \mathbb{R}^2 : x = 0\right\}, \quad S_2 = \left\{(x, y) \in \mathbb{R}^2 : x > 0, y = \sin \frac{1}{x}\right\}.$$

Show that S is topologically connected.

Exercise 2 (10 points)**.** Let Λ be an abst4ract index set and let $\{S_\alpha\}_{\alpha \in \Lambda}$ be a collection of convex sets in $\mathbb{R}^n$. Show that the intersection $\bigcap_{\alpha \in \Lambda} S_\alpha$ is convex.

Exercise 3 (10 points)**.** Let C be a convex set in $\mathbb{R}^n$, show that its closure $\overline{C}$ and its interior $\operatorname{int}(C)$ are also convex.

Exercise 4 (10 points)**.** Let C be a convex set in $\mathbb{R}^n$ and let $A \in \mathbb{R}^{m \times n}$ and $\boldsymbol{b} \in \mathbb{R}^m$. Show that the set $AC + b := \{A\boldsymbol{x} + b : \boldsymbol{x} \in C\}$ is also a convex set in $\mathbb{R}^m$.

Exercise 5 (10 points)**.** Let S be any set in $\mathbb{R}^n$. Show that its convex hull $\operatorname{conv}(S)$ is identical to the intersection of all convex sets containing S.