## GEOMETRY (701939001, 751764001, 113-2) - HOMEWORK 6

Return to TA by: May 27, 2025 (Tuesday) 16:00

Total marks: 50

**Exercise 1** (10+10 points). We consider the twisted cubic described by the parameterization  $\alpha(t) = (x(t), y(t), z(t))$  with

$$x(t) = t$$
,  $y(t) = t^2$ ,  $z(t) = \frac{2}{3}t^3$ .

Compute its curvature  $\kappa$  and its torsion  $\tau$ .

**Exercise 2.** Consider the  $C^{\infty}$  curve

$$\boldsymbol{\alpha}(t) = \begin{cases} (t, 0, e^{-1/t^2}) &, t > 0, \\ (t, e^{-1/t^2}, 0) &, t < 0, \\ (0, 0, 0) &, t = 0. \end{cases}$$

- (a) (10 points). Show that  $\alpha'(t) \neq 0$  for all  $t \in \mathbb{R}$ .
- (b) (10 points). Show that the curvature  $\kappa(t) \neq 0$  for all  $t \neq 0$  but  $\kappa(0) = 0$ .
- (c) (10 points). Show that the torsion  $\tau$  can be defined so that  $\tau(t) = 0$  for all  $t \in \mathbb{R}$  even though  $\alpha$  is not a plane curve.