

DIFFERENTIAL EQUATIONS (751873001, 113-1) - HOMEWORK 1

Return by September 26, 2024 (Thursday) 23:59

Total marks: 50

Special requirement. All homeworks must be prepared by using L^AT_EX.

Exercise 1 (15 points). Verify that the initial-value problem

$$u'(t) = (u(t))^{1/3} \text{ for all } t \in \mathbb{R}, \quad u(t_0) = 0$$

has at least two nontivial $C^1(\mathbb{R})$ -solutions:

$$u(t) = \begin{cases} 0 & , t \leq t_0, \\ \left(\frac{2}{3}(t - t_0)\right)^{3/2} & , t > t_0, \end{cases}$$

and

$$u(t) = \begin{cases} 0 & , t \leq t_0, \\ -\left(\frac{2}{3}(t - t_0)\right)^{3/2} & , t > t_0. \end{cases}$$

Exercise 2 (15 points). Verify that the initial-value problem

$$u'(t) = \sqrt{|u(t)|} \text{ for all } t \in \mathbb{R}, \quad u(t_0) = 0$$

has at least one nontivial $C^1(\mathbb{R})$ -solution:

$$u(t) = \begin{cases} -\frac{1}{4}(t - t_0)^2 & , t \leq t_0, \\ \frac{1}{4}(t - t_0)^2 & , t > t_0. \end{cases}$$

Exercise 3 (10 points). Show that the initial value problem

$$u'(t) = \frac{1}{(3 - (t - 1)^2)(9 - (t - 5)^2)} \text{ for all } t \in (1 - \sqrt{2}, 1 + \sqrt{2}),$$
$$u(1) = 5,$$

has a unique $C^1((1 - \sqrt{2}, 1 + \sqrt{2}))$ -solution.

Exercise 4 (10 points). Show that the initial value problem

$$u'(t) = \frac{1}{(1 + (t - 4)^2)(5 + (t - 3)^2)} \text{ for all } t \in \mathbb{R},$$
$$u(4) = 3,$$

has a unique $C^1(\mathbb{R})$ -solution.