

DIFFERENTIAL EQUATIONS (751873001, 113-1) - HOMEWORK 3

Return by October 31, 2024 (Thursday) 23:59

Total marks: 50

Special requirement. All homeworks must be prepared by using L^AT_EX.

Exercise 1 (15 points). Solve the initial value problem

$$xu\partial_x u - \partial_y u = 0, \quad u(x, 0) = x.$$

Exercise 2 (15 points). Solve the initial value problem

$$x\partial_y u - y\partial_x u = u, \quad u(x, 0) = h(x)$$

provided that $h \in C^1(\mathbb{R})$.

Exercise 3 (5+5+5 points). Let $A, B \in \mathbb{C}^{n \times n}$, show that

- (a) $\|A\| = (\operatorname{tr}(A^*A))^{1/2}$, where A^* is the *conjugate transpose* (or *adjoint*) of $A \in \mathbb{C}^{n \times n}$.
- (b) $\|A + B\| \leq \|A\| + \|B\|$,
- (c) $\|AB\| \leq \|A\|\|B\|$.

Exercise 4 (5 points). Show that A_m converges to A if and only if $\lim_{m \rightarrow \infty} \|A_m - A\| = 0$.