DIFFERENTIAL EQUATIONS (751873001, 114-1) - HOMEWORK 1

Return by September 19, 2025 (Friday) 23:59

Total marks: 50

Special requirement. All homeworks must be prepared by using LaTeX.

Exercise 1 (15 points)**.** Verify that the initial-value problem

$$u'(t) = (u(t))^{1/3} \text{ for all } t \in \mathbb{R}, \quad u(t_0) = 0$$

has at least two nontivial $C^1(\mathbb{R})$-solutions:

$$u(t) = \begin{cases} 0 & , t \leq t_0, \\ \left(\dfrac{2}{3}(t - t_0)\right)^{3/2} & , t > t_0, \end{cases}$$

and

$$u(t) = \begin{cases} 0 & , t \leq t_0, \\ -\left(\dfrac{2}{3}(t - t_0)\right)^{3/2} & , t > t_0. \end{cases}$$

Exercise 2 (15 points)**.** Verify that the initial-value problem

$$u'(t) = \sqrt{|u(t)|} \text{ for all } t \in \mathbb{R}, \quad u(t_0) = 0$$

has at least one nontivial $C^1(\mathbb{R})$-solution:

$$u(t) = \begin{cases} -\frac{1}{4}(t - t_0)^2 & , t \leq t_0, \\ \frac{1}{4}(t - t_0)^2 & , t > t_0. \end{cases}$$

Exercise 3 (10 points)**.** Show that the initial value problem

$$\begin{aligned} &u'(t) = \frac{1}{(3 - (t-1)^2)(9 - (u-5)^2)} \text{ for all } t \in (1 - \sqrt{2}, 1 + \sqrt{2}), \\ &u(1) = 5, \end{aligned}$$

has a unique $C^1((1 - \sqrt{2}, 1 + \sqrt{2}))$-solution.

Exercise 4 (10 points)**.** Show that the initial value problem

$$\begin{aligned} &u'(t) = \frac{1}{(1 + (t-4)^2)(5 + (u-3)^2)} \text{ for all } t \in \mathbb{R}, \\ &u(4) = 3, \end{aligned}$$

has a unique $C^1(\mathbb{R})$-solution (in this case, we call it the “global” solution).