

DIFFERENTIAL EQUATIONS (751873002, 113-2) - HOMEWORK 3

Return by March 20, 2025 (Thursday) 23:59

Total marks: 50

Special requirement. All homework must be prepared by using L^AT_EX.

Exercise 1 (10 points). Show that each normed space $(X, \|\cdot\|)$ is also a metric space.

Exercise 2 (10 points). Verify that the form $d_{\mathcal{D}_K}(\cdot, \cdot)$ given by (3.1.3) is a metric, see Exercise 3.1.10.

Exercise 3 (10 points). Let Ω be an open set in \mathbb{R}^n . Show that $(\mathcal{D}(\Omega), d_{\mathcal{D}(\Omega)}(\cdot, \cdot))$ is not complete by verify the counterexample in Exercise 3.1.13.

Exercise 4 (10 points). Prove that for every $c \in \mathbb{R}$ one has

$$(e^{-c|x|})' = -ce^{-cx}H(x) + ce^{cx}H(-x) \text{ in } \mathcal{D}'(\mathbb{R}).$$

Exercise 5 (10 points). Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x \ln |x| - x & \text{for } x \neq 0, \\ 0 & \text{for } x = 0. \end{cases}$$

Prove that f is a continuous function and compute its distributional derivative f' .