# PARTIAL DIFFERENTIAL EQUATIONS (701925001, 751944001, 112-2) - HOMEWORK 3

Return by: March 21, 2024 (Thursday) 16:00

Total marks: 50

**Note.** One should try to solve all problems in the lecture note. Here I only choose some of them in this homework.

**Exercise 1** (10 points)**.** Given any $f \in C^1(\mathbb{R})$, solve the equation $(1+t^2)\partial_t u + \partial_x u = 0$ with $u(0,x) = f(x)$ and identify the range of $x$.

**Exercise 2** (10 points)**.** Given any $f \in C^1(\mathbb{R})$, solve the equation $t\partial_t u + x\partial_x u = 0$ with $u(0,x) = f(x)$ and identify the range of $x$.

**Exercise 3** (10 points)**.** Solve the equation $x\partial_t u + t\partial_x u = 0$ with $u(0,x) = e^{-x^2}$.

**Exercise 4** (10 points)**.** Let $n = 2$. Classify each of the equations:

(1) $\partial_1^2 u - 5\partial_1\partial_2 u = 0$;
(2) $4\partial_1^2 u - 12\partial_1\partial_2 u + 9\partial_2^2 u + \partial_2 u = 0$;
(3) $4\partial_1^2 u + 6\partial_1\partial_2 u + 9\partial_2^2 u = 0$.

**Exercise 5** (10 points)**.** Classify each of the equations:

(1) $-\Delta u + \boldsymbol{b}\cdot\nabla u + cu = 0$ with $u = u(\boldsymbol{x})$.
(2) **Heat equation (or caloric equation, diffusion equation)** $\partial_t u - \Delta u + \boldsymbol{b}\cdot\nabla u + cu = 0$ with $u = u(t,\boldsymbol{x})$.
(3) **Wave equation** $\partial_t^2 u - \Delta u + \boldsymbol{b}\cdot\nabla u + cu = 0$ with $u = u(t,\boldsymbol{x})$.