STATISTICS "FOR DEPARTMENT OF MATHEMATICAL SCIENCES" (701007001, 114-2) - HOMEWORK 1

Return by April 8, 2026 (Wednesday) 15:00

Total marks: 50

Note. Using LaTeXto prepare your homework is encouraged but not required. If you do so, please print it out and submit a hard copy.

0.3524	0.6534	0.5702	0.0141	0.3636	0.1891	0.7294	0.1531	0.7034	0.6875
0.5850	0.9779	0.7725	0.7359	0.7771	0.1433	0.2303	0.5903	0.1553	0.4583

TABLE 1. $N = 20$ samples (round to 4 digits) generated from uniform distribution on $(0, 1)$

Exercise 1. We consider the samples in Table 1.

(a) (**5 points**) Estimating the expectation of uniform distribution on $(0, 1)$ by (unbiased) sample mean

$$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} X_i.$$

(b) (**5+5+5 points**) Estimating the expectation of uniform distribution on $(0, 1)$ by the truncated mean

$$\overline{X}_{\text{tr}} := \frac{1}{N-2m+2}\sum_{i=m}^{N-m+1} \max\{X_1, \cdots, X_i\}$$

with $m = 3$ (i.e. truncated proportion $\frac{m-1}{N} = \frac{1}{10}$), with $m = 5$ (i.e. truncated proportion $\frac{m-1}{N} = \frac{1}{5}$) and with $m = 11$ (i.e. truncated proportion $\frac{m-1}{N} = \frac{1}{2}$)

(c) (**5 points**) Estimating the variance of uniform distribution on $(0, 1)$ by variance of samples (which is biased, but consistent)

$$S_b^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \overline{X})^2.$$

(d) (**5 points**) Estimating the variance of uniform distribution on $(0, 1)$ by unbiased sample variance.

$$S_u^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2.$$

(e) (**5 points**) Estimating the standard deviation of uniform distribution on $(0, 1)$ by the square root of unbiased sample variance (which is biased, but consistent)

$$S_u = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2}.$$

(f) (**5 points**) Estimating the standard deviation of uniform distribution on $(0,1)$ by unbiased standard deviation, given by $K_N S_u$, where

$$K_N := \sqrt{\frac{N-1}{2}}\frac{\Gamma(\frac{N-1}{2})}{\Gamma(\frac{N}{2})} = \overbrace{\sqrt{\frac{N-1}{2}}\exp\left(\ln\Gamma\left(\frac{N-1}{2}\right) - \ln\Gamma\left(\frac{N}{2}\right)\right)}^{\text{more numerically stable for large } N},$$

and Γ is the Gamma function.

1.2153	0.7722	2.5223	2.0187	3.8605	2.4625	2.9747	0.1639	0.8423	4.3666
4.7683	1.0266	0.9891	0.7111	4.4883	4.889	1.8089	3.6891	3.1977	4.4926

TABLE 2. $N = 20$ samples (round to 4 digits) generated from uniform distribution on $(0,\theta)$ for some unknown θ

Exercise 2. We consider the samples in Table 2. It is known that estimating θ by the largest element 4.8890 is biased.

(a) (**5 points**) Estimating θ by $\hat{\theta}_u := \frac{N+1}{N}\max_{1\le i\le N} X_i$.

(b) (**5 points**) Estimating θ by $\hat{\theta} := 2\overline{X}$.