STATISTICS "FOR DEPARTMENT OF MATHEMATICAL SCIENCES"
(701007001, 114-2) - HOMEWORK 3

Return by April 22, 2026 (Wednesday) 15:00

Total marks: 50

Note. Using LaTeXto prepare your homework is encouraged but not required. If you do so, please print it out and submit a hard copy.

Exercise 1 (10+10 points)**.** Suppose that $X_1, \cdots, X_N$ are random samples from a distribution depends on parameters α and β, with

$$\mathbb{E}X_i = \alpha\beta \quad \text{and} \quad \mathbb{E}X_i^2 = \beta^2(\alpha+1)\alpha.$$

(a) Propose an estimator $\hat{\theta} = \hat{\theta}(X_1, \cdots, X_N)$ of the unknown parameter $\theta = \frac{\alpha}{\beta}$, and prove that such estimator is consistent, that is, $\hat{\theta} \to \theta$ a.s. as $N \to \infty$.

(b) Propose an estimator $\hat{\theta} = \hat{\theta}(X_1, \cdots, X_N)$ of the unknown parameter $\theta = e^{\alpha}$, and prove that such estimator is consistent, that is, $\hat{\theta} \to \theta$ a.s. as $N \to \infty$.

Exercise 2 (10+10 points)**.** Let $X_1, \cdots, X_N$ be random samples drawn from the uniform distribution on $(0, \theta)$, i.e.,

$$\mathbb{P}(X_i \le y) = \begin{cases} 0 & , y \le 0, \\ y/\theta & , 0 \le y \le \theta, \\ 1 & , y \ge \theta. \end{cases}$$

(a) Propose an estimator $\hat{\theta} = \hat{\theta}(X_1, \cdots, X_N)$ of the unknown parameter θ, and prove that such estimator is consistent, that is, $\hat{\theta} \to \theta$ a.s. as $N \to \infty$.

(b) Propose an estimator $\hat{\theta} = \hat{\theta}(X_1, \cdots, X_N)$ of the unknown parameter θ, and show that such estimator is minimal sufficient.

Exercise 3 (10 points)**.** Let $X_1, \cdots, X_N$ be random samples drawn from the uniform distribution on $(\theta, \theta+1)$, i.e.,

$$\mathbb{P}(X_i \le y) = \begin{cases} 0 & , y \le \theta, \\ y-\theta & , \theta \le y \le \theta+1, \\ 1 & , y \ge \theta+1. \end{cases}$$

Propose a statistic T of θ and show that it is sufficient (**Hint.** 2-dimensional statistics).