STATISTICS "FOR DEPARTMENT OF MATHEMATICAL SCIENCES" (701007001, 114-2) - HOMEWORK 4

Return by May 27, 2026 (Wednesday) 15:00

Total marks: 50

Note. Using LaTeXto prepare your homework is encouraged but not required. If you do so, please print it out and submit a hard copy.

Exercise 1. A company seeks to determine the optimal warranty period for a model of heat pumps. In order to do so, a random sample of $N = 15$ heat pumps yielded the observations in Table 1. Assume that the lifetime distribution is exponential (Definition~2.6.9).

$x_1 = 2.0$	$x_2 = 1.3$	$x_3 = 6.0$	$x_4 = 1.9$	$x_5 = 5.1$
$x_6 = 0.4$	$x_7 = 1.0$	$x_8 = 5.3$	$x_9 = 15.7$	$x_{10} = 0.7$
$x_{11} = 4.8$	$x_{12} = 0.9$	$x_{13} = 12.2$	$x_{14} = 5.3$	$x_{15} = 0.6$

TABLE 1. Lifetime of heat pumps x_i (in years)

(a) (10 points). Determine the warranty period so that no more than 5% of customers are expected to file a claim, by constructing a one-sided 95% confidence interval for the expected lifetime.
(b) (10 points). Construct a one-sided 99% confidence interval for the expected lifetime.
(c) (10 points). What is a 95% confidence interval for the variance of the lifetime distribution? (**Note.** The definition of a confidence interval depends on the choice of point estimator. I am willing to use a biased estimator, provided it can be shown to be consistent as $N \to \infty$)

Exercise 2. A study by Uetake on individuals' ability to walk in a straight line, reported in the paper[1] "Can We Really Walk Straight?", provides accompanying data on cadence (strides per second) for a sample of $N = 20$ randomly selected healthy men, as shown in Table 2. Assume that the population distribution of cadence is (approximately) normal.

$x_1 = 0.95$	$x_2 = 0.85$	$x_3 = 0.92$	$x_4 = 0.95$	$x_5 = 0.93$
$x_6 = 0.86$	$x_7 = 1.00$	$x_8 = 0.92$	$x_9 = 0.85$	$x_{10} = 0.81$
$x_{11} = 0.78$	$x_{12} = 0.93$	$x_{13} = 0.93$	$x_{14} = 1.05$	$x_{15} = 0.93$
$x_{16} = 1.06$	$x_{17} = 1.06$	$x_{18} = 0.96$	$x_{19} = 0.81$	$x_{20} = 0.96$

TABLE 2. Lifetime of heat pumps x_i (in years)

(a) (10 points). Calculate and interpret a 95% confidence interval for population mean cadence.
(a) (10 points). Calculate and interpret a 95% prediction interval for the cadence of a single individual randomly selected from this population.

[1] https://doi.org/10.1002/ajpa.1330890104